

3-D Face Analysis and Identification Based on Statistical Shape Modelling

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Abstract

This paper presents an effective method of statistical shape representation for automatic face analysis and identification in 3-D. The method combines statistical shape modelling techniques and the non-rigid deformation matching scheme. This work is distinguished by three key contributions. The first is the introduction of a new 3-D shape registration method using hierarchical landmark detection and multilevel B-spline warping technique, which allows accurate dense correspondence search for statistical model construction. The second is the shape representation approach, based on Laplacian Eigenmap, which provides a nonlinear submanifold that links underlying structure of facial data. The third contribution is a hybrid method for matching the statistical model and test dataset which controls the levels of the model's deformation at different matching stages and so increases chance of the successful matching. The proposed method is tested on the public database, BU-3DFE. Results indicate that it can achieve extremely high verification rates in a series of tests, thus providing real-world practicality.

1 Introduction

Face-recognition biometrics provides greater **convenience and flexibility** than existing conventional approaches for personal identification (e.g. DNA and fingerprint analysis, iris recognition). The face is one of widely used biometric features and has advantage over the others, such as natural, contactless and nonintrusive. Because of the advance in imaging technology and ever increasing computing power, face recognition research has received significant attention, especially during the past two decades. It has also been used for applications in crime prevention and detection [1, 2].

Automatic face recognition is always a challenging task. Facial appearances can vary substantially from one individual to another due to difference in head pose, lighting conditions and facial expression. Also differences in appearance of the same person can be extremely variable (see [3] for example), while differences in appearance between different people can be less variable [4]. Broadly speaking, there are three major challenges which are often addressed in the research of face recognition, including facial data acquisition, facial feature

representation and face identification. Facial data acquisition looks into how to acquire, detect and located facial data in complex scenes. Facial feature representation concerns the extraction of features or patterns which could be used for representing identify of faces under different scenarios. Face identification concentrates mainly on finding suitable classification methodologies to allocate the corresponding identity to the face based on the extracted facial features. The current study focuses on the second challenge which seeks robust methods for the facial features extraction.

According to the types of facial feature used for the face recognition, the methodologies can be generally classified into two major categories: original-data based and feature based. The original-data based approaches process the face as a single unity and make use of the entire face region in order to compute similarity. A number of works have been done for face recognition by directly using 2-D facial images [5]; while some have been carried out based on 3-D facial data [6]. In contrast to the original-data based methods, the feature based methods focus on the local features or areas on the face, such as landmark, curve and region. Landmark feature based methods detect representative key facial points often associated with biological meaning in order to construct feature space. Shi et al. [7] introduced a method based on the so called 'soft' landmark - that is, the landmarks that are easily located on actual skin surfaces, such as eye corners, mouth corners, nose edge, etc. These methods found that these landmarks vary significantly if different people derive them. Curve feature based methods extract discriminative surface curves for facial representation. The symmetric profile curve from the intersection between the symmetry plane and the 3-D facial scans was described in [8]. Three facial curves which intersect the facial scan using horizontal and vertical planes as well as a cylinder were proposed in [9]. Region feature based methods are similar to landmark feature based. They use dense point-clouds from specific regions rather than sparse feature points for the face recognition. Queirolo et al. [10] used a union of the segmented regions from faces. These regions include the circular nose area, elliptical nose area and the upper head. Regions segmented by the median and Gaussian curvature were utilised for the face identification in [11].

The statistical approach has been one of the most successful and well-studied techniques for face recognition over the past few decades. It can be used in conjunction with

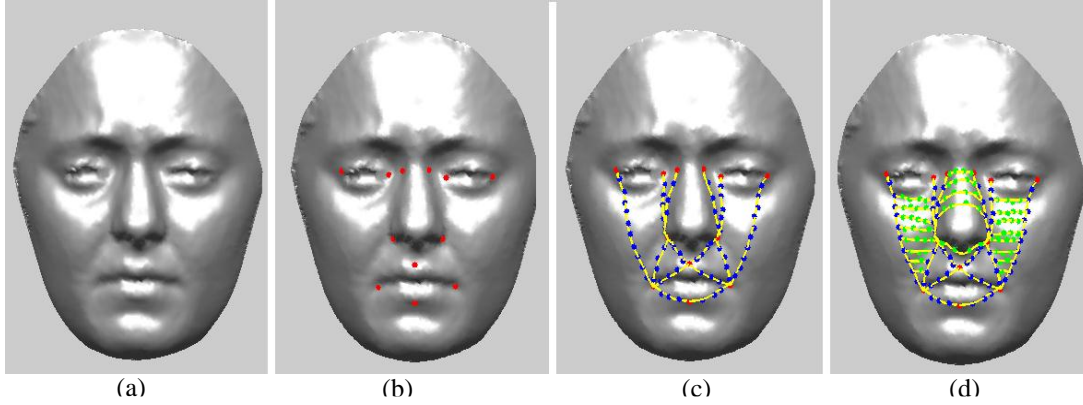


Figure 1. Example of hierarchical landmark detection: (a) original face; (b) key landmarks; (c) first group landmarks in blue; (d) second group landmarks in green.

both original data and feature-based methods. In this paper, an effective statistical shape modelling scheme is proposed for 3-D face representation and recognition, which is a combination of the statistical model and the non-rigid deformation shape registration process. Instead of the widely used models, Principal Component Analysis (PCA) and Linear Discriminant Analysis (LDA), Laplacian Eigenmap (LE) is chosen for this work. As LE aims preserve local structure of the data which often links to the non-linear structure of the manifold, it can handle the variation of facial expression for face recognition - often a major source of variance in the data. In order to make sure the statistical model would be constructed correctly for a set of control faces (i.e. the training faces), hierarchical facial landmark detection is performed first. This is followed by point-cloud registration which captures both rigid and non-rigid deformation with the aim of multilevel B-spline warping function. A hybrid version of the Iterative Closest Point (ICP) method [12] with the newly introduced modifications is applied to match the models to test faces. The proposed approach promises to handle large variation in facial expression and head pose.

The remainder of the paper is organised as follow: Section 2 introduces the hierarchical facial landmark detection. Section 3 describes the dense correspondence search for the control dataset. Section 4 explains the statistical shape model, LE. Section 5 illustrates the model matching based on the hybrid ICP algorithm with modifications. The comprehensive experimental results using the statistical shape models for face recognition are presented in Section 6. Finally, concluding remarks and future work are given in Section 7.

2 Landmark detection

The registration of 3-D faces in the control dataset is essential for accurate construction of statistical shape models. Since the control faces are variable by identity and facial expression a relatively dense facial landmark set is needed to model change of facial shape. In this work, the landmarks are detected using a hierarchical approach that consists of two stages, namely key landmark extraction and geodesic-based sampling. The former method starts with locating a number of landmarks that can be found on most 3-D faces, which are

insensitive across the population, facial expression and pose. Using the approach proposed by Dorai and Jain [13], a set of six key landmarks can be extracted using the shape index: two inner eye corners, two outside eye corners and two mouth corners. In combination with another facial landmark detection methods using the profile of facial symmetry plane [14], an additional six key landmarks can be identified that are located on the upper nose base, nose corners, and upper and lower extremities of lip. Figure 1(b) demonstrates the location of all 12 key landmarks extracted from a 3-D face.

Once key landmarks are located, more can be identified hierarchically using the geodesic-based sampling technique. The concept here is to generate extra landmarks based on the pairs of detected landmark that have already been obtained. This is done by computing the geodesic paths and sampling the paths with equal geodesic length between a pair. The geodesic distance and the corresponding path between any pair of detected landmarks are computed based on the fast march method. As the geodesic measurement preserves intrinsic properties of the shape, it is invariant to facial expressions. Two extra groups of landmarks can be identified in addition to the key landmark set. The first group of landmarks are established based on the key landmarks using geodesic sampling as shown in Figure 1(c). The second group are constructed based on the extracted landmarks from the first group as shown in Figure 1(d). The resulting set has 154 landmarks in total that cover **the inner facial regions** which are prone to the change of facial shapes due to the population and expression. They include key landmarks (12 points), first group landmarks (66 points) and second group landmarks (76 points), to provide sufficient information for data registration in the statistical shape model construction. By applying hierarchical landmark detection to all of the control faces, landmark correspondences are established.

3 Deformation alignment

Registration of 3-D faces in the control dataset is important for statistical model construction because inaccurate registration results in erroneous dense correspondences. Since the facial geometry and deformation of the control faces are different, a non-rigid deformable registration problem needs to be resolved. A procedure based on multilevel B-splines

warping and closest point matching is proposed here in which the registration is completed with the following four steps. The first is to select a template face with a neutral expression from control dataset so that it can be used for aligning in general. The second step is to establish a warping function ϕ from the corresponding landmark sets from the template face and any of the control faces using the multilevel B-spline technique. The third step is to use the warping function ϕ to transfer points on template face to align them with the control face. The final step is to compute the dense correspondences between template and control faces using the measurement of closest distance. By repeating steps two to four on the other control faces, the entire control dataset is aligned and registered.

Multilevel B-splines was first proposed by Lee et al. [15]. This approach uses a coarse-to-fine hierarchy of control lattices to generate a sequence of bicubic B-spline functions in order to approximate the desired interpolation functions. In this work, the approach is adopted in order to estimate an optimal free-form deformation (FFD) according to corresponding landmarks from any given pair of 3-D faces, namely source and target. The FFD is then applied to all 3-D points on the source in order to precisely warp with the target.

Given a moving set of landmarks (source) $\mathbf{P} = (\mathbf{p}_1, \mathbf{p}_1, \dots, \mathbf{p}_L)$ and a fixed set of landmarks (target) $\mathbf{Q} = (\mathbf{q}_1, \mathbf{q}_2, \dots, \mathbf{q}_L)$, where $\mathbf{p}_k = (p_{xu}, p_{yu}, p_{zu})$ and $\mathbf{q}_k = (q_{xu}, q_{yu}, q_{zu})$ denote x, y and z coordinates of the u -th corresponding pair and L is the total number of corresponding landmarks, the objectives of the multilevel B-splines warping algorithm is first to estimate the FFD based on a set of displacement vectors $\mathbf{D} = \mathbf{P} - \mathbf{Q}$, which is associated with the target. A transformation of B-spline functions is calculated to interpolate the set of displacement vector, and is defined as,

$$T(x, y, z) = \sum_{i=0}^3 \sum_{j=0}^3 \sum_{k=0}^3 B_i(r) B_j(s) B_k(t) \phi_{i+m+j, n+k} \quad (1)$$

where

$$l = \left\lfloor \frac{p_x}{\delta} \right\rfloor - 1, \quad m = \left\lfloor \frac{p_y}{\delta} \right\rfloor - 1, \quad n = \left\lfloor \frac{p_z}{\delta} \right\rfloor - 1 \quad (2)$$

and

$$r = \frac{p_x}{\delta} - \left\lfloor \frac{p_x}{\delta} \right\rfloor, \quad s = \frac{p_y}{\delta} - \left\lfloor \frac{p_y}{\delta} \right\rfloor, \quad t = \frac{p_z}{\delta} - \left\lfloor \frac{p_z}{\delta} \right\rfloor \quad (3)$$

B_i, B_j, B_k represent the B-spline basis function which define the contribution of each control point based on its distance from the landmark. The general form of these basis functions can be expressed as,

$$\begin{aligned} B_0(t) &= (1-t)^3 / 6 \\ B_1(t) &= (3t^3 - 6t^2 + 4) / 6 \\ B_2(t) &= (-3t^3 + 3t^2 + 3t + 1) / 6 \\ B_3(t) &= t^3 / 6 \end{aligned} \quad (4)$$

ϕ is a rectangular set of control points with uniform spacing δ , which is defined as a bounding box of all landmarks as shown

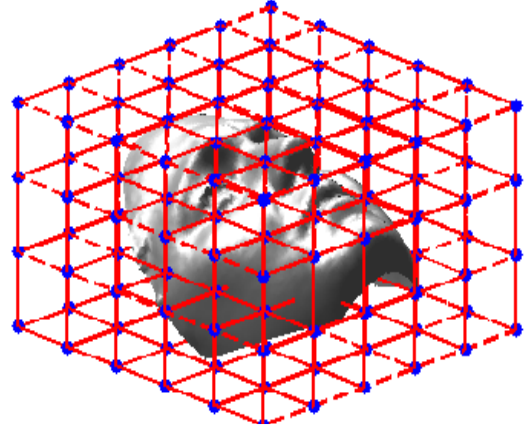


Figure 2. Source face and the rectangular grid of B-spline control points in blue.

in Figure 2. Using the least square approximation, the control point set ϕ are computed and applied to warp all points of the source to the target. Because of the locality of B-splines, it is worth noticing that each point in the source is affected by its neighbouring set of 64 points. In order to solve the problem of control point spacing and achieve good approximation as well as high smoothness, a coarse-to-fine hierarchy of control point generation was applied [15].

4 Statistical model

When statistical models are used for face analysis and recognition, PCA and LDA are often the popular choice, to produce compact representation based on the dimensionality of linear manifolds. However, these models fail to discover the underlying nonlinear structure of facial data especially for the faces containing expressions [16]. The Laplacian Eigenmap (LE) method is able to handle a wider range of data variability while preserving local structure, which often links to the nonlinear structure of the manifold. LE therefore appears to be a good candidate for face verification and recognition even under the influence of changing expression.

LE is a graph based non-linear technique with isometric mapping from original space to reduced low-dimensional space. The optimality criterion to minimise is,

$$\frac{1}{2} \sum_{i,j} (\mathbf{a}_i - \mathbf{a}_j)^2 S_{ij}, \quad i, j = 1, 2, \dots, N \quad (5)$$

\mathbf{a}_i and \mathbf{a}_j is the i -th and j -th linear transformations that are defined as,

$$\mathbf{a}_k = \mathbf{W}^T \mathbf{x}_k \quad k = 1, 2, \dots, N \quad (6)$$

$\mathbf{W} = \{\mathbf{w}_1, \mathbf{w}_2, \dots, \mathbf{w}_k\}$ is a matrix with orthonormal set of vectors representing the eigenvectors of the control faces, \mathbf{x}_k is a given set of N control faces $\{\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_N\}$, and S_{ij} is the adjacency matrix, which defines the weighted similarity graph for all connected vertexes, can be computed as,

$$S_{ij} = \exp \left(-\frac{\|\mathbf{x}_i - \mathbf{x}_j\|^2}{2\delta} \right) \quad (7)$$

δ is calculated as

$$\delta = \frac{1}{N} \sum_{i=1}^N \min_{j \neq i} \|\mathbf{x}_i - \mathbf{x}_j\|^2 \quad (8)$$

\mathbf{x}_i and \mathbf{x}_j is the i -th and j -th control face, respectively. The K-nearest neighbour method can also be applied to retain k edges for each \mathbf{x}_i and remove other connections. Derived from (14), the objective function can be rewritten as,

$$\arg \min_{\mathbf{W}^T XDX^T \mathbf{W} = \mathbf{I}} \mathbf{W}^T X L X^T \mathbf{W} \quad (9)$$

$X = \{\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_N\}$ contains all control faces, $L = D - S$ with each entry of D is calculated as $d_{ii} = \sum_{j=1}^N S_{ij}$ and $d_{ij} = 0$ for all $i \neq j, \forall d \in D$.

Proposition. The objective function can be rewritten in the following short form where $\mathbf{a} \in \mathbb{R}^n$,

$$\mathbf{W}^T X L X^T \mathbf{W} = \frac{1}{2} \sum_{i,j} (\mathbf{a}_i - \mathbf{a}_j)^2 S_{ij} \quad (10)$$

Proof.

$$\begin{aligned} & \frac{1}{2} \sum_{i,j} (\mathbf{a}_i - \mathbf{a}_j)^2 S_{ij} \\ &= \frac{1}{2} \sum_{i,j} (\mathbf{W}^T \mathbf{x}_i - \mathbf{W}^T \mathbf{x}_j)^2 S_{ij} \\ &= \sum_{i,j} \mathbf{W}^T \mathbf{x}_i S_{ij} \mathbf{x}_i^T \mathbf{W} - \sum_{i,j} \mathbf{W}^T \mathbf{x}_i S_{ij} \mathbf{x}_j^T \mathbf{W} \\ &= \frac{1}{2} \sum_i \mathbf{W}^T \mathbf{x}_i D_i \mathbf{x}_i^T \mathbf{W} + \frac{1}{2} \sum_j \mathbf{W}^T \mathbf{x}_j D_j \mathbf{x}_j^T \mathbf{W} - \sum_{i,j} \mathbf{W}^T \mathbf{x}_i S_{ij} \mathbf{x}_j^T \mathbf{W} \\ &= \mathbf{W}^T X D X^T \mathbf{W} - \mathbf{W}^T X S X^T \mathbf{W} \\ &= \mathbf{W}^T X L X^T \mathbf{W} \end{aligned} \quad (11)$$

The objective function is solved by the eigenvector corresponding to the smallest eigenvalue of the generalised eigenvalue problem,

$$X L X^T \mathbf{W} = \lambda X D X^T \mathbf{W} \quad (12)$$

Obtained from (14) the edge weights are all non-negative, which indicates $\mathbf{W}^T X L X^T \mathbf{W} \geq 0$. This proves that L is positive semidefinite, along with D . Therefore $X L X^T$ and $X D X^T$ are both semidefinite and symmetrical.

5 Model matching

For the purpose of face analysis and recognition, the low dimensional feature vectors need to be extracted from test faces. The standard procedure for doing this is to project test faces to the feature space that is created using the control faces. To ensure that the projection is accurate, the dense point correspondences from the statistical model to test faces must be established by model matching. The model matching is a process based on the hybrid version of ICP [12] that iteratively estimates the shape and pose parameters in turn. While pose parameters control the location and position of the models, shape parameters encapsulate deformation in the matching. The objective function for model matching is to minimise the following cost function,

$$E(\mathbf{a}, \mathbf{R}, \mathbf{t}) = \|\mathbf{S}_T - \mathbf{S}_M\|^2 = \|\mathbf{S}_T - \mathbf{R}(\mathbf{W}_{opt} \mathbf{a} + \mathbf{t})\|^2 \quad (13)$$

where \mathbf{R} and \mathbf{t} are the rotation matrix and translation vector, respectively, \mathbf{S}_M is generally the statistical model, \mathbf{S}_T denotes the test face, \mathbf{W}_{opt} is the optimal eigenvectors and \mathbf{a} is the

corresponding weight/feature vectors which control the shape representation of the models. In order to solve for all unknown parameters, the following standard optimisation scheme should be followed [17]:

1. Align model and test face by using their centroid points, to minimise translation.
2. The ICP algorithm is applied to solve for the rotation and translation parameters and estimate the dense correspondences between model and test face.
3. Given the rotation matrix, translation vectors and dense correspondence in step 2, minimise the cost function E by solving feature vector \mathbf{a} using back-projection described as,

$$\mathbf{a} = \mathbf{W}_{opt}^T \mathbf{x} \quad (14)$$

where \mathbf{x} is the estimated dense correspondences for a test face.

4. Generate a new instance of the statistical model using the feature vector \mathbf{a} and repeat steps 2 to 4 until pre-set convergence condition is reached.

In this work, a hierarchical strategy is applied to improve the standard model matching approach for 3-D data. It provides a coarse-to-fine strategy which allows a smooth and gradual change in the model's shape. The strategy introduces two modifications: multi-resolution correspondence search and multi-level model deformation.

5.1 Multi-resolution correspondences search

The search for dense correspondences is the most time-consuming step in any ICP type method. In order to speed up this process, and make it less prone to local minimum convergence, a multi-resolution scheme is developed that starts searching for the correspondences from a coarse level, and gradually increases the density in a series of finer resolutions as registration error reduces. Random sampling is used to sub-sample the data to obtain lower resolution data in the proposed method. Four levels are used for the implementation with each reducing the number of points by one quarter. Based on the test using the BU-3DFE database the algorithm with the proposed correspondence search saved at least 15% of computation time on average; it also improved alignment results by avoiding local minima.

5.2 Multilevel model deformation

The number of eigenvectors and the size of the feature vectors play an important role in the matching process. The common way to determine how many eigenvectors are used depends on the shape variation that can be modelled by these eigenvectors, for example using 90% of the total shape variation of all control faces. Therefore the number of eigenvectors used is fixed prior to the model matching. Using the fixed number of the eigenvectors can usually provide a reasonable final result of matching if a good initial alignment is used. Otherwise, it may lead to the minimization of the cost function (during model refinement) towards a local minimum. This can be explained by the fact that when a large number of eigenvectors are used, the statistical models have a high

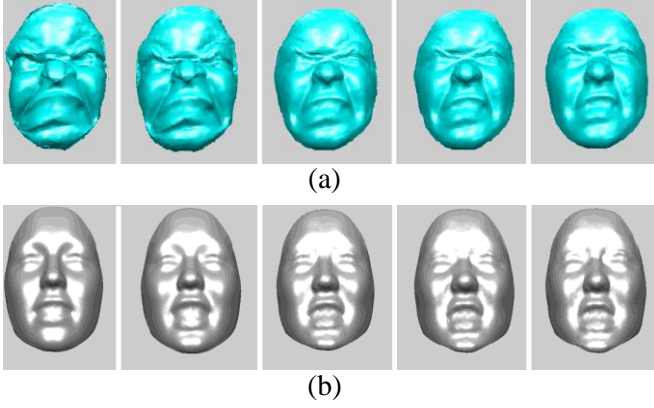


Figure 3. Example of intermediate results obtained during the iteration of the model matching: (a) fixed number of eigenvectors; (b) multilevel model deformation.

degree of freedom enabling it to iteratively deform to a shape representing a local minimum, if that happened, the models matching were instantiated in the basing of this minimum and produces an inappropriate result as shown in Figure 3(a). On the other hand, models with a small number of eigenvectors have a low degree of freedom which constraints the deformation and prevents it from converging to shapes responsible for local minima of the cost function. However it also limits the ability of the model to accurately represent the data. In order to solve this problem, multilevel model deformation is employed as this uses an adaptive number of eigenvectors. The matching procedure starts with a small number of eigenvectors. Although the registration error can be very large at this stage, the algorithm is able to provide a rough approximation of the data. When registration error is reduced, the number of the eigenvectors can be gradually increased to provide more shape flexibility and allow the model to match the data. This adaptive number eigenvector can be represented as:

$$\alpha_k = \mathbf{W}_{optk}^T \mathbf{x}_k \quad (15)$$

where \mathbf{W}_{optk} and α_k is the adaptive eigenvectors and feature vector, k indicates the level of the matching, and \mathbf{x}_k is the estimated dense correspondences for a test face. A few examples of the intermediate results obtained using the multilevel model deformation are shown in Figure 3(b). It can be seen that the multilevel deformation model not only bestows a smooth transition between iterations of the model but also enables the model to closely approximate the data.

6 Experimental results

To demonstrate the effectiveness of the proposed methods for face verification and recognition, a well-known public 3-D facial database, BU-3DFE [18] was utilised for our evaluation. A variety of pattern classifiers could be applied to the face recognition, including, Nearest-Neighbour, Naive Bayesian, Support Vector Machine, etc. **As the focus of this paper is on the facial shape modelling rather than the design of best possible classification algorithm,** one of the simplest classifiers, Nearest-Neighbour classifier is chosen in this study. The Euclidean metric is used as the distance measure.

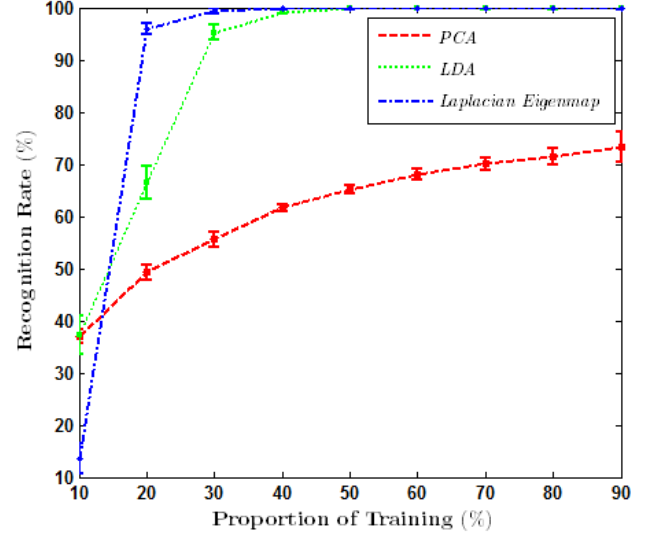


Figure 4. Recognition accuracy versus the training to testing ratio

In order to investigate the effect of the training to testing ratio on face recognition, a series of experimental tests were run that used different proportions of training and testing data are performed. The experiment used all 2,500 3-D faces that are available from the BU-3DFE database. It started with the training to testing ratio at 1/9, i.e., using 10% of overall 3-D faces as the training and 90% of overall faces as the testing, and gradually increased the portion of the training to 20%, 30%, etc. In the same way, the corresponding fraction of the testing was reduced to 80%, 70% and so on. In order to have a comprehensive measurement, the 3-D faces were divided into ten subsets. Each subset contained all 100 people with various expressions randomly selected. During the experiment one or more of the subsets is chosen for training while the remaining is used for testing. Such experiments are repeated at least ten times for each training to testing ratio, with the different subsets selected as the training each time.

The results of face recognition based on the variation of training and testing ratio are shown in Figure 4. As can be seen, all three statistical models significantly improve face recognition as the proportion of the training dataset is increased. The average maximum recognition that can be achieved by the PCA-based approach is $74\% \pm 3\%$ when 90% of faces from the database are used for training. Compared with the PCA-based approach, the LDA-based and LE-based methods both reach the highest recognition rate at 100%. It is worth noting that the LE-based method obtains 100% accuracy by only using 30% of all faces available for training, which is 10% less than the LDA-based approach.

Coping with varying facial expressions is always a great challenge for face verification and recognition, and is unavoidable in practical situations. Therefore, the ability to handling expression changes is becoming a standard criterion for measuring how good a face recognition algorithm is. In order to test robustness to facial expression changes for the three statistical models, a special protocol was designed for the extreme case, for one kind of expression versus another, in which the faces with one kind of expression were used for training and the faces with another type of expression for

testing. Since there are seven kinds of expression available in the BU-3DFE database, one neutral and six universal types, the experiment was repeated 42 times. The experimental results show that the LDA-based approach performed comparably to LE-based methods, while PCA-based performed poorly. The average recognition for PCA, LDA and LE were 55.2%, 90.1% and 93.7%, respectively. Table 1 shows the recognition results of LE in detail. It was found that the LE-based method can handle most expressions extremely well by achieving over 90% recognition only using faces with one specific type of expressions. Although few of them have lower recognition, they are still reasonable as the lowest recognition rate was 76% when our special protocol was used.

<i>Train./Test.</i>	<i>Neu.</i> (%)	<i>Ang.</i> (%)	<i>Disg.</i> (%)	<i>Fear</i> (%)	<i>Happ.</i> (%)	<i>Sad.</i> (%)	<i>Surp.</i> (%)
Neutral	-	95.0	90.1	92.9	96.1	96.2	83.2
Anger	98.9	-	91.1	90.0	93.4	98.6	78.1
Disgust	92.8	97.6	-	99.1	98.9	97.3	92.7
Fear	95.8	97.3	99.7	-	99.3	97.3	94.2
Happiness	93.2	78.1	81.5	91.6	-	89.0	99.2
Sadness	96.1	98.2	93.7	90.6	94.0	-	80.7
Surprise	96.3	76.2	78.5	90.0	98.1	88.2	-

Table 1: Recognition performance of LE by expression.

7 Conclusion

An effective statistical shape modelling scheme for face analysis and recognition has been presented in this paper. It used hierarchical landmark detection and multilevel B-spline warping to help an accurate search of dense point correspondences for the control faces, and ICP-based hybrid model matching method for the alignment of statistical model and faces from testing dataset. From the experimental results based on the BU-3DFE database, it can be seen that proposed LE model appears to be robust at handling face identification under various scenarios in contrast with PCA and LDA. The research can be extended further by considering the practical application. One possible extension of the work is to evaluate the generalisability of the new algorithm to **different databases** that have been captured with different devices and in different environments. Further, the issue of missing data could be resolved by modifying the statistical shape modelling scheme as it is one of the most challenging part in the real-world face recognition.

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